IOTA Working Group 2 Diuscussion Start-up paper for OSC

Optical Stochastic Cooling Experiment at IOTA

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Besides the experiments on highly non-linear integrable optics, the 150 - 200 MeV electron storage ring IOTA will be used to carry out a test of the optical stochastic cooling (OSC) technique. This method is a novel approach to the beam dynamics and has potential serious implications for a range of heavier (than electron) particle accelerators, ranging from LHC and RHIC for hadron and heavy ion colliders and other rings. Accelerator experts have called for an experimental demonstration of OSC for a long time.

The experiment will have two phases. At the first step the cooling will be achieved without an optical amplifier. It should introduce a damping rate higher than the cooling rate due to synchrotron radiation. At the second phase, an optical amplifier will be used.

The IOTA facility will offer unique opportunity to carry out the proposed research toward demonstration of the feasibility of the optical stochastic cooling technique. That research requires a dedicated storage ring (IOTA) and its operation with 100-150 MeV electrons. It cannot be carried out anywhere else as there are no existing electron storage rings in that energy range which can afford installation of special insertions (optical equipment, wigglers, etc.), and offers special arrangements of the optics lattice and precise control of the insertion devices and the ring elements. Previous attempts to identify such an existing facility were unsuccessful (e.g., the proposal to use the MIT-Bates storage ring was found to be very expensive as the ring nominal energy and size were significantly beyond what was needed for the OSC demonstration).

Introduction

The stochastic cooling suggested by Simon Van der Meer [1] and further developed by [2, 3, 4, 5, 6,] has been successfully used in a number of machines for particle cooling and accumulation. However it was not helpful for cooling of bunched beams in proton-(anti)proton colliders due to very high phase density of the bunches. Only recently, bunched beam stochastic cooling has been introduced in operation at the Relativistic Heavy Ion Collider (RHIC) [7]. In the case of optimal cooling the maximum damping rate can be estimated as:

$$\lambda \approx \frac{2W\sigma_s}{N\,C} \quad ,$$

where W is the bandwidth of the system, N is the number of particles in the bunch, σ_s is the rms bunch length, and C is the machine circumference. For the LHC proton beam ($\sigma_s = 9$ cm, $C = 26.66$ km) and a system with one octave bandwidth and its upper boundary of 8 GHz one obtains λ^{-1}=12000 hour. An effective cooling requires faster damping rates by least 3 orders of magnitude. The OSC suggested by Mikhailichenko, Zolotorev, and Zholents [7,8] can have a bandwidth of ~10^{14} Hz and, thus, suggests a way to achieve required damping rates. The basic principles of the

OSC are similar to the normal (microwave) stochastic cooling except that it uses optical frequencies, allowing an increase of system bandwidth by 4 orders of magnitude.

In the OSC a particle radiates EM radiation in the pickup wiggler. Then, the radiation amplified in an optical amplifier makes a longitudinal kick to the particle in the kicker wiggler as shown in Figure 1. Further we will call these wigglers pickup and kicker. A magnetic chicane is used to make space for an optical amplifier and to bring the particle and the radiation together in the kicker wiggler. In further consideration we assume that the path lengths of particle and radiation are adjusted so that the relative particle momentum change is equal to:

$$\delta p / p = -\kappa \sin\left(k \Delta s\right) \quad . \tag{1}$$

Here $k = 2\pi/\lambda$ is the radiation wave number, and Δs is the particle displacement on the way from the pickup wiggler to the kicker wiggler relative to the reference particle which obtains zero kick:

$$\Delta s = M_{51} x + M_{52} \theta_x + M_{56}\left(\Delta p / p\right) \tag{2}$$

Here M_{5n} are the elements of 6x6 transfer matrix from pickup to kicker, x, θ_x and $\Delta p/p$ are the particle coordinate, angle and relative momentum deviation in the pickup.

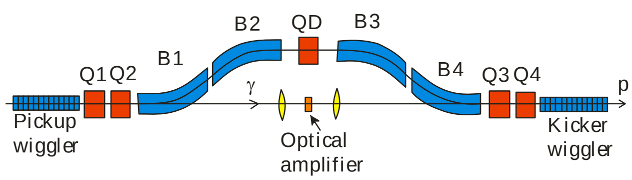

Figure 1: Schematic of Optical Stochastic Cooling (OSC).

For small amplitude oscillations the horizontal and vertical cooling rates are [9]:

$$\begin{bmatrix} \lambda_x \\ \lambda_s \end{bmatrix} = \frac{k\kappa}{2} \begin{bmatrix} M_{56} - C\eta_{pk} \\ C\eta_{pk} \end{bmatrix} , \tag{3}$$

where $\eta_{pk} = \left(M_{51} D_p + M_{52} D_p' + M_{56}\right) / C$ is the partial momentum compaction determined so that for a particle without betatron oscillations and with momentum deviation $\Delta p/p$ the longitudinal displacement relative to the reference particle on the way from pickup and kicker is equal to $C\eta_{pk}$ $\Delta p/p$. Here we also assume that there is no x-y coupling. Introduction of x-y coupling outside the cooling area allows redistribution of horizontal damping rate into both transverse planes. The sum of damping rates, $\Sigma\lambda_n = k\kappa M_{56}/2$, does not depend on the beam optics outside of the cooling chicane.

An increase of betatron and synchrotron amplitudes results in a decrease of damping rates [9]:

$$\begin{aligned} \lambda_x(a_x, a_s) &= F_x(a_x, a_s)\lambda_x \quad , \\ \lambda_s(a_x, a_s) &= F_s(a_x, a_s)\lambda_s \quad , \end{aligned} \tag{4}$$

where the form factors are:

$$F_x(a_x, a_s) = 2J_0(a_s)J_1(a_x)/a_x\,,$$
$$F_s(a_x, a_s) = 2J_0(a_x)J_1(a_s)/a_s\,; \tag{5}$$

and a_x and a_s are the amplitudes of longitudinal particle motion due to betatron and synchrotron oscillations expressed in the units of e.-m. wave phase:

$$\begin{aligned} a_x &= k\sqrt{\varepsilon_1\left(\beta_p M_{51}{}^2 - 2\alpha_p M_{51}M_{52} + \left(1+\alpha_p{}^2\right)M_{52}{}^2\right)}, \\ a_s &= kC\left|\eta_{pk}\right|(\Delta p/p)_{max} \end{aligned} \tag{6}$$

Here ε_1 is the Courant-Snyder invariant of a particle, and $(\Delta p/p)_{max}$ is its maximum momentum deviation. As one can see from Eqs. (4) and (5), the damping rate changes sign if any of amplitudes exceeds the first root of the Bessel function $J_0(x)$, $a_{x,}a_s > \mu_0 \approx 2.405$.

The following conclusions can be drawn from Eqs. (3) and (6). M_{56} depends only on focusing inside the chicane, while η_{pk} additionally depends on the dispersion at the chicane beginning, *i.e.* on the optics in the rest of the ring. Consequently, the damping rates ratio,

$$\lambda_x / \lambda_s = M_{56} / C\eta_{pk} - 1 \tag{7}$$

and the longitudinal cooling range,

$$n_{\sigma s} \equiv (\Delta p/p)_{max} / \sigma_p = \mu_0 / |kC\eta_{pk}\sigma_p|\,, \tag{8}$$

depend on focusing and dispersion inside the chicane, but do not depend on the beta-function. Here σ_p is the relative rms momentum spread. In contrast, the transverse cooling range,

$$n_{\sigma x} \equiv \frac{\varepsilon_{max}}{\varepsilon} = \frac{\mu_0{}^2/\left(k^2\varepsilon\right)}{\beta_p M_{51}{}^2 - 2\alpha_p M_{51}M_{52} + \left(1+\alpha_p{}^2\right)M_{52}{}^2}\,, \tag{9}$$

does not depend on the dispersion but depends on the beta-function. Here ε is the rms transverse emittance.

Below we consider two cooling schemes. The first one is passive cooling [10] where radiation is focused into the kicker wiggler but is not amplified; and the second one is active where an optical amplifier is used. Both of them have its advantages and drawbacks. In the case of passive cooling one does not need an amplifier and, consequently, can use higher optical frequencies and larger bandwidth which boost the gain. It also requires smaller path difference which considerably increases the cooling ranges, $n_{\sigma s}$ and $n_{\sigma x}$. In the case of an active system one can reduce the length and magnetic field of the wigglers, but it requires an additional delay in the chicane to compensate a delay in the optical amplifier (~5 mm). Making an amplifier at required power and wavelength can be a challenging problem too.

Beam Optics

The main parameters of the ring, called IOTA [9], are shown in the Table I. The OSC system will take one of four straight sections with length of ~5 m. The beta-function and dispersion in the section are presented in Figure 2. The optics was built for 800 nm radiation where an optical amplification is a feasible task. The following limitations were taken into account in the optics design. The chicane should separate the radiation and the beam by 40 mm making a sufficiently large separation between the electron beam and optical amplifier. The cooling ranges, $n_{\sigma s}$ and $n_{\sigma x}$, (before the OSC is engaged) have to be large enough so that the major fraction of the beam would be cooled. The path length difference acquired by electron beam in the chicane has to be large enough to compensate delay in optical amplifier. Note that the rectangular dipoles do not produce horizontal focusing. Therefore in the absence of other focusing inside chicane the partial slip factor is equal to M_{56}/C and does not depend on the dispersion. Consequently, there is no transverse cooling. To achieve it a defocusing quad was introduced in the chicane center. The strength of this quad is limited by reduction of transverse cooling range, $n_{\sigma x}$ which requires sufficiently large dispersion in the chicane. The major parameters of the cooling section are presented in Table 2.

Table 1: Main Parameters of IOTA storage ring configured for OSC.

Circumference	38.7 m
Nominal beam energy	150 MeV
Bending field	7 kG
Betatron tune	3.5 ÷ 7.2
Maximum β-function	3 ÷ 9 m
Transverse vertical emittance, non-normalized	3 nm r.m.s
Rms momentum spread, σ_p	$1.5\cdot10^{-4}$
SR damping rates (ampl.), $\lambda_s / \lambda_\perp$	4 / 2 s^{-1}

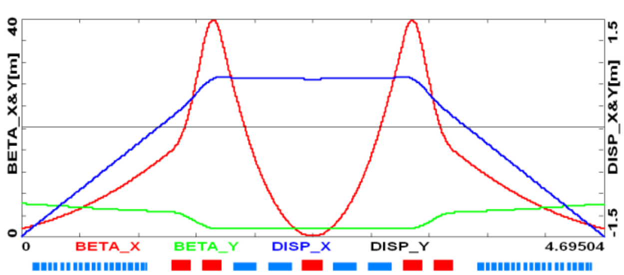

Figure 2: Optical lattice functions in the OSC section.

Table 2: Major parameters of chicane beam optics.

M_{56}	8.7 mm
Cooling rates ratio, λ_x/λ_s	7.5
Horizontal beam separation	40 mm
Delay in the chicane	4.5 mm
Cooling ranges (before OSC), $n_{\sigma x}/n_{\sigma s}$	3.5 / 2
Dipole magnetic field	4 kG
Dipole length	18 cm
Strength of central quad, $\int GdL$	1.52 kG
Strength of central quad, $\int GdL$	1.52 kG

The rms emittance and momentum spread are comparatively large for the chosen wavelength of 800 nm. To accommodate it the optics was tuned to maximize the cooling ranges. In particular, we choose (1) the large cooling rates ratio to increase $n_{\sigma s}$, and (2) small beta-function in the chicane center (2 cm) to increase $n_{\sigma x}$. That resulted in high sensitivity of cooling parameters. Simulations show that relative accuracies should be ~1% for the horizontal beta-function, ~2 cm for the dispersion, and ~2% for the focusing of central quadrupole (see Figure 3).

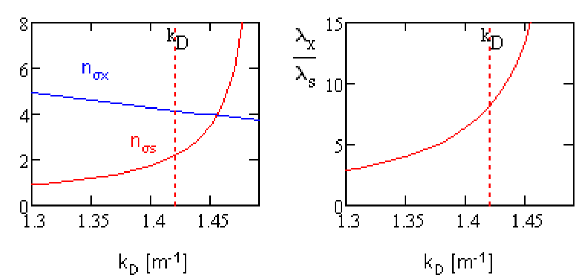

Figure 3: Dependencies of cooling ranges (left) and ratio of damping rates on focusing strength of central quadrupole.

Light Optics

Let a particle move in the flat undulator so that its coordinates depend on time as following:

$$\mathrm{v}_x = c\theta_e \sin\tau' , \qquad \mathrm{v}_y = 0 ,$$
$$\mathrm{v}_z = c\left(1 - \frac{1}{2\gamma^2} - \frac{\theta_e^{\,2}}{2}\sin^2\tau'\right), \; \tau' = \omega_u t' + \psi , \tag{10}$$

where γ is the particle relativistic factor (γ>> 1), and ω_u is the frequency of particle motion in the undulator. Substituting velocities of Eq. (10) to the Liénard-Wiechert formula [12] for the horizontal component of electric field in the far zone one obtains:

$$E_x(r,t) = 4e\omega_u\gamma^4\theta_e\cos\tau' \times$$
$$\frac{1+\gamma^2\left(\theta^2\left(1-2\cos^2\phi\right) - 2\theta\theta_e\sin\tau'\cos\phi - \theta_e^{\,2}\sin^2\tau'\right)}{cR\left(1+\gamma^2\left(\theta^2 + 2\theta\theta_e\sin\tau'\cos\phi + \theta_e^{\,2}\sin^2\tau'\right)\right)^3}, \tag{11}$$

where θ and ϕ are the angles for the vector from the radiation point, $\mathbf{r}'$, to the observation point, $\mathbf{r}$, in the polar coordinate system, $R = \left|\mathbf{r} - \mathbf{r}'\right|$, and $t - t' = R/c$. In further calculations we will be leaving the radiation in the first harmonic only,

$$E_\omega(r) = \frac{\omega(\theta)}{\pi}\int_0^{2\pi/\omega(\theta)} E_x(r,t)\,\mathrm{e}^{-i\omega t}\,dt, \quad ,$$
$$\omega(\theta) = 2\gamma^2\omega_u / \left(1+\gamma^2\left(\theta^2 + \theta_e^{\,2}/2\right)\right), \tag{12}$$

assuming that the radiation of higher harmonics is absorbed in the lenses and/or not amplified by optical amplifier. Then taking into account delay in the lens and applying Kirchhoff formula,

$$E(r'') = \frac{1}{2\pi i c}\int_S \frac{\omega(\theta)E_\omega(r)}{\left|r''-r\right|}\mathrm{e}^{i\omega\left|r''-r\right|}\,ds , \tag{13}$$

one obtains the electric field in the focal point. For a large acceptance lens, $\theta_m \geq \theta_e + 3/\gamma$, located in the middle of pickup-to-kicker distance the results of numerical integration can be interpolated by the following equation:

$$E_x = 4e\omega_u^{\,2}\gamma^4\theta_e F(\gamma\theta_e)/\left(3c^2\right),$$
$$F(K) \approx 1/\left(1 + 2.15K^2 + 1.28K^4\right), \quad K \leq 4 , \tag{14}$$

where $K = \gamma\theta_e$ is the undulator parameter, and θ_m is the lens angular size from the radiation point. Integrating the force along the kicker length one obtains the longitudinal kick amplitude:

$$c\delta p_{\max} \equiv \kappa cp = 2e^4 B_0^{\,2}\gamma^2 L F(K)/\left(3m^2c^4\right). \tag{15}$$

The bandwidth of the system is much more narrow (≤10%) if an optical amplifier is used. In this case the kick value is:

$$c\delta p_{\max} = \frac{2e^4 B_0^{\,2} \gamma^4 L \theta_{\mathrm{m}}^{\,2}}{m^2 c^4 \left(1+K^2\right)^3} = \frac{2e^4 B_0^{\,2} \gamma^4 L}{m^2 c^4 \left(1+K^2\right)^3} \frac{\Delta\omega}{\omega} \ , \tag{16}$$

where in the second equality we assumed that $\gamma^2 \Delta\theta_m^{\,2} = \left(1+K^2\right)\Delta\omega/\omega$.

Above we assumed that the radiation coming out from the pickup is focused at the particle location throughout the entire course of the particle motion in the kicker. This can be achieved if the distance to the lens is much larger than the length of wiggler—a condition which is impossible to fully achieve in practice. A practical solution can be obtained with lens telescope which has the transfer matrix from the center of pickup to the center of kicker equal to ±**I**, where **I** is the identity matrix. The simplest telescope has 3 lenses as shown in Figure 4. For symmetrically located lenses their focusing distances are:

$$F_1 = \frac{LL_1}{L+L_1}, \quad F_2 = \frac{L_1^{\,2}}{2\left(L+L_1\right)} \quad . \tag{17}$$

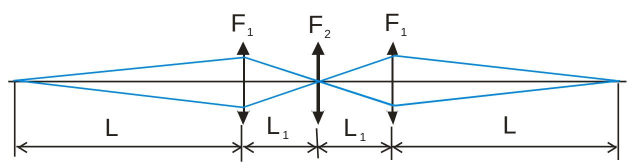

Figure 4: Light optics layout for passive cooling.

Table 3 presents the main parameters of undulators, light optics, and OSC damping rates for the passive and active OSC. The passive cooling requires about one octave band (0.8-1.6 μm). The wave packet lengthening looks satisfactory for 4.5 mm light delay in magnesium fluoride. However a suppression of transverse focusing chromaticity looks to be an extremely challenging problem and needs additional study. Combination of glasses with normal and abnormal dispersions might be a good direction for study. A Ti: Sapphire optical amplifier is considered a good candidate capable of delivering ~20 dB gain within the allocated signal delay. Technical details are presently under study.

The parametrs for the Optical Stochastic Cooling esxperiment in IOTA are rapisly evolving and will converge as we approach the experimental realization in IOTA soon.

Table 3: Main parameters of Optical Stochastic Cooling (OSC).

Undulator parameter, K	1.5
Undulator period	6.53 cm
Number of periods	14
Total undulator length	0.915 m
Distance between undulators	3.6 m
Telescope length, $2L_1$	0.25 m
Telescope aperture, 2a	40 mm
Lens focal distances, F_1 / F_2	116 / 4.3 mm
Damping rates of passive OSC(x/y/s)	100/100/25 s^{-1}
Damp. rates 20 dB gain & 10% band	300/300/75 s^{-1}

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